Benchmark of different CFL conditions for IMPES

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Abstract

The IMplicit Pressure Explicit Saturation (IMPES) method is a prevalent way to simulate multiphase flows in porous media. The numerical stability of this sequential method implies limitations on time step which may depend on the flow regime studied. In this note, three stability criteria related to the IMPES method, that differ in their construction on the observed variables, are compared on homogeneous and heterogeneous configurations for different two-phase flow regimes (viscous/capillary/gravitational). This highlights there is no single optimal criterion always ensuring stability and efficiency. For capillary dominated flows, the Todd's condition is the most efficient while the standard Coat's condition should be preferred for viscous flows. When gravity effects are present, the Coat's condition must be restricted but remains more efficient than the Todd's condition.

L'IMplicit Pressure Explicit Saturation method (IMPES) est l'une des principales mthodes pour traiter les cas d'coulements multiphasiques en milieux poreux. La stabilit numrique de cette mthode squentielle implique des contraintes diffrentes sur le pas de temps selon le rgime d'coulement tudi. Dans cette note, les trois principaux critres de stabilit lis l'IMPES sont tests sur des milieux homognes et htrognes pour diffrents rgimes (visqueux/capillaire/gravitaire). Cette tude montre qu'aucun critre optimal, runissant stabilit et efficacit, ne se dgage. Pour les coulements capillaires, la condition de Todd est la plus efficace tandis que la condition standard de Coats est prfrable pour les coulements visqueux. Quand les effets gravitaires sont pris en compte, la condition de Coats doit tre restreinte mais demeure plus efficace que la condition de Todd.

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1. Introduction

Among the possible numerical methods used to simulate two-phase flow in porous media [1,2,3], the IMplicit Pressure Explicit Saturation (IMPES) method remains in use today [4,5]. This sequential algorithm, originally proposed by Sheldon [6], has the advantage to substantially reduce the size of linear

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systems to solve, compared to a fully implicit method. In return, the method is limited by important numerical stability restrictions on the time step size. Hybrid methods, Adaptive Implicit Methods (AIM), have also been proposed and treat implicitly unknowns in regions with high throughput ratio[24].

The numerical instabilities are due to the non-linear effects involved in two-phase flow in porous media and mainly related to capillary pressure and relative permeability laws. The explicit resolution of saturation requires the linearization of capillary and permeability laws which could lead to numerical instability. This can lead to erroneous calculations of the saturation field and, in the worst cases, to the end of the simulation (the computed saturation is out of the limits). The various laws and their complexity make stability even harder to predict and, therefore, different stability criteria have been proposed and studied. Todd [7] has first derived a condition based on averaged spatial and temporal saturation variation, which provides an increasing/decreasing factor for the time step. Coats [8], through a proper Von Neumann analysis, has derived a CFL criteria based on mobility related terms, fluxes and capillary pressure. One can also use the classical CFL [9] condition to ensure stability. Other stability studies have been conducted focusing on upstream scheme [25], on switching criteria for AIM [24,27] or on extension to compositional and black-oil models [26]. Even if the Coats' stability criterion is commonly used, it may be very restrictive in certain circumstances and is therefore not necessarily the optimum choice. To our knowledge, there is no study in the literature comparing these different stability criteria to highlight their effectiveness for different porous media viscous flow regimes involving, or not, capillary and gravitational effects.

This need for numerical stability is all the more important as two-phase flow in porous media are often subject to physical instabilities. This class of instabilities can be caused by various configurations such as counter current flows and layered flows or by properties of the studied system (mobility ratio, viscosity ratio, permeability distribution). The most commonly known and studied instability is the viscous fingering phenomenon [10]. When one is interested in simulating this kind of physical instability, the numerical stability should be ensured to avoid the perturbation of the system by a numerical artifact.

In a recent work [11], the IMPES method has been implemented and developed in the open-source framework OpenFOAM [12,13]. This open-source implementation has been successfully employed in various fields, such as two-phase flow in structured bed packing [14] and waste management [15].

The scope of the paper is the performance benchmark of existing criteria taken from the literature. A methodology is set up to compare their efficiency in terms of computational cost (number of linear solver iterations) for various cases. This study has been designed for helping IMPES users that struggle with stability issues in choosing the most suited criteria for their simulation. Its ambition is not to develop a new criteria but to gather user experiences on different configuration with different criteria. We proposed to cross-compare Todd, Coats and classical Courant-Friedrichs-Lewy criteria listed above, for different flow regimes (viscous/capillary/gravitational) and for homogeneous and heterogeneous permeability fields without singularities (e.g., no wellbore model).

This note is organized in two parts. In the first one, two-phase flow equations for porous media are described, detailing the IMPES algorithm and presenting the three stability numbers investigated. We introduce mathematical formulation of the different criteria, stating which phenomena are included in the theoretical form. In the second part, numerical experiments are performed to explore the different stability conditions on three classical configurations, and define their efficiency.

2. Two-phase flow and stability numbers

2.1. *Mathematical model*

Two-phase flows under investigation are assumed incompressible, viscous and isothermal. The wetting and non-wetting phases are respectively denoted w and n. The mass conservation equation for each phase reads

$$\phi \frac{\partial S_w}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{u}_w = q_w,$$
$$-\phi \frac{\partial S_w}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{u}_n = q_n, \tag{1}$$

with the obvious relationship

$$S_w + S_n = 1. \tag{2}$$

In these equations, S_α refers to the saturation, ϕ is the porosity, q_α is the mass source/sink term and $\mathbf{u}_\alpha$ denotes the superficial velocity for each phase α. The latter are slow enough to be modeled by generalized Darcy's laws [16],

$$\mathbf{u}_w = -\boldsymbol{K} \cdot \lambda_w \left(\boldsymbol{\nabla} p_n - \rho_w \mathbf{g} - \boldsymbol{\nabla} p_c \right),$$
$$\mathbf{u}_n = -\boldsymbol{K} \cdot \lambda_n \left(\boldsymbol{\nabla} p_n - \rho_n \mathbf{g} \right), \tag{3}$$

where $\boldsymbol{K}$ is the permeability tensor intrinsic to the porous material, ρ_α is the fluid density and $\mathbf{g}$ the gravitational acceleration. The capillary pressure, p_c, i.e. the pressure difference between both phases depends on the saturation [17] and reads,

$$p_c \left(S_w \right) = p_n - p_w. \tag{4}$$

The mobility λ_α is defined as

$$\lambda_\alpha = \left(\frac{k_{r,\alpha}(S_w)}{\mu_\alpha} \right)_{\alpha=w,n}, \tag{5}$$

where μ_α is the fluid viscosity and $k_{r,\alpha}$ is the relative permeability function.

Many models exist in the literature to represent the capillary pressure and the relative permeabilities according to the saturation [17,18,19,20,21]. In the present study, the well-established Brooks and Corey model [19] is used. With such model, capillary pressure, p_c, and relative permeabilities, $k_{r,\alpha}$, read

$$p_c \left(S_w \right) = p_{c,0} \, S_e^{-\frac{1}{m}}$$
$$k_{r,n} \left(S_w \right) = k_{r,n_{max}} \left(1 - S_e \right)^{\frac{3m+2}{m}} \tag{6}$$
$$k_{r,w} \left(S_w \right) = k_{r,w_{max}} S_e^{\frac{3m+2}{m}}.$$

where $p_{c,0}$, $k_{r,n_{max}}$ and $k_{r,w_{max}}$ are model parameters and the pore-size index, m, is a characteristic number of the porous medium considered: small for large range pore-size distribution, large for relatively uniform pore-size distribution. The reduced saturation,

$$S_e = \left(\frac{S_w - S_{w,irr}}{1 - S_{w,irr} - S_{n,res}} \right), \tag{7}$$

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represents that amount of wetting phase that can flow. It depends on the irreducible wetting saturation, $S_{w,irr}$ and the residual non-wetting saturation, $S_{n,res}$.

2.2. IMPES algorithm

The chosen unknowns for the numerical implementation are the pressure of the non-wetting phase and the saturation of the wetting phase (p_n, S_w). The saturation S_w is governed by the wetting phase mass conservation Eq. (1) and the pressure p_n satisfies the global mass conservation,

$$\boldsymbol{\nabla} \cdot (-\boldsymbol{K} \cdot \lambda_t \boldsymbol{\nabla} p_n) - \boldsymbol{\nabla} \cdot (\boldsymbol{K} \cdot \Psi (\rho_w - \rho_n) \cdot \mathbf{g} - \boldsymbol{K} \cdot \Psi p_c' \boldsymbol{\nabla} S_w) = q_t, \tag{8}$$

where $\lambda_t = \lambda_w + \lambda_n$ is the total mobility, $q_t = q_w + q_n$ is the total sink/source term and $\Psi = \frac{2\lambda_w \lambda_n}{\lambda_w + \lambda_n}$ is the harmonic average of mobilities. The first derivative of capillary pressure with respect to the wetting phase saturation S_w is introduced as $|p_c'|$.

The IMPES solution algorithm consists in solving implicitly the pressure equation (8) and explicitly the saturation equation (1). The details of the implemented algorithm can be found in a previous work [11]. In the following simulations, a first order upwind interpolation is used for mobility related terms and a backward Euler scheme is adopted for time discretization.

Linear solver used in the experiment is a conjugate gradient solver with a diagonal incomplete Cholesky preconditioner. It is a commonly used pair when dealing with symmetric matrices. Generalized geometric-algebraic multi-grid solver might be an appropriate alternative for solving this equation over large domains. As cases treated in the next section remain simple in term of number of cells and considering that solver efficiency is not in the scope of this work, the choice of such a preconditioner-solver pair is not disadvantageous.

2.3. Stability criteria

In this section the three tested CFL conditions ensuring the stability of IMPES simulations are described: namely the classic Courant number condition (Co), the Todd's derived number condition (T) and the Coats' derived number condition (C). For each criterion, a time-step factor F, which gives the increase or decrease in the time-step size during the simulation, is defined.

Classic Courant number condition (Co)
This condition limits the Courant number of each phase α, by a user defined value, Co_{max}:

$$Co = \frac{1}{2} \max_{i,\alpha} \left(\frac{\sum\limits_{faces \subset i} |q_{\alpha,f}|}{V_i} \Delta t \right) < Co_{max} \qquad i = 1, N_{cells}. \tag{9}$$

This Courant number is a direct adaptation of the classical one extended to two-phase flows [9]. It involves the sum of absolute values of fluxes in phase α through every faces of cell i (term $\sum\limits_{faces \subset i} |q_{\alpha,f}|$) and the volume V_i of the cell i. It is designed to ensure stability of the hyperbolic saturation equation . The time-step factor F is defined as:

$$F = \frac{Co_{max}}{Co}. \tag{10}$$

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Todd's number condition (T)

The first stability criterion dedicated to the IMPES algorithm [7] has been derived taking into account the discretized form of the pressure and saturation equations. It leads to a constraint on the time step, split into two time-step restrictions, regarding if capillary pressure p_c, or relative permeabilities k_r, are considered

$$\Delta t \leq \min_i \left[\Delta t_{p_c,i}, \Delta t_{k_r,i} \right] \qquad i = 1, N_{cells}. \tag{11}$$

Capillary restriction on time-step can be expressed as

$$\Delta t_{p_c,i} \leq \frac{\phi V_i}{|p_c'| \sum\limits_{faces \subset i} (T_f \Psi)} \qquad i = 1, N_{cells}. \tag{12}$$

where $T_f = (KA/\Delta x)$ is the transmissivity of face f, whose area is noted A and whose distance to the cell center is noted Δx. Harmonic interpolation of K is chosen for computing the transmissivity T_f. Equation (12), reformulated as a CFL condition, reads :

$$CFL_{Todd,pc} = \Delta t_{p_c,i} \frac{|p_c'| \sum\limits_{faces \subset i} (T_f \Psi)}{\phi V_i} < CFL_{Todd,max}, \quad i = 1, N_{cells}. \tag{13}$$

which introduces a user-defined upper limit $CFL_{Todd,max}$.

Relative permeabilities restriction, formulated in terms of inter-cell fluxes, reads

$$\Delta t_{k_r,i} \leq \frac{\phi V_i}{f_{w,i}' \left(\sum\limits_{faces \subset i} |q_f| \right)} \qquad i = 1, N_{cells}, \tag{14}$$

with f_w' the derivative of the fractional flow $f_w = \frac{\lambda_w}{\lambda_t}$ with respect to the saturation of the wetting phase S_w and q_f the total flux through the f face. In terms of spatial and temporal saturation variation, a time step ratio can be used, with the 1/2 factor depending on the chosen spatial discretization scheme (e.g. here 1-pt upwind in 1D):

$$T = \frac{\Delta t_{k_r}^{n+1}}{\Delta t_{k_r}^n} = \frac{1}{2} \frac{\frac{1}{N_{cells}} \sum\limits_i |\Delta_{i,i+1} S_w|}{\max\limits_i (|\Delta_t S_w|)} \qquad i = 1, N_{cells}. \tag{15}$$

Here, the equation (15) defines directly a time step factor referred in the following as T number. The symbol $\Delta_{i,i+1} S_w$ stands for the difference between two neighbor cells and $\Delta_t S_w$ is the saturation difference between n and $n-1$ time state. The time-step factor F includes both parts, capillary pressure and relative permeability, and is defined as:

$$F = min(T, \frac{CFL_{Todd,max}}{CFL_{Todd,pc}}). \tag{16}$$

Coats' number condition (C)

More recently, starting from inequality (12) and (14) using Neumann's stability analysis, a new stability number C has been developed [8]:
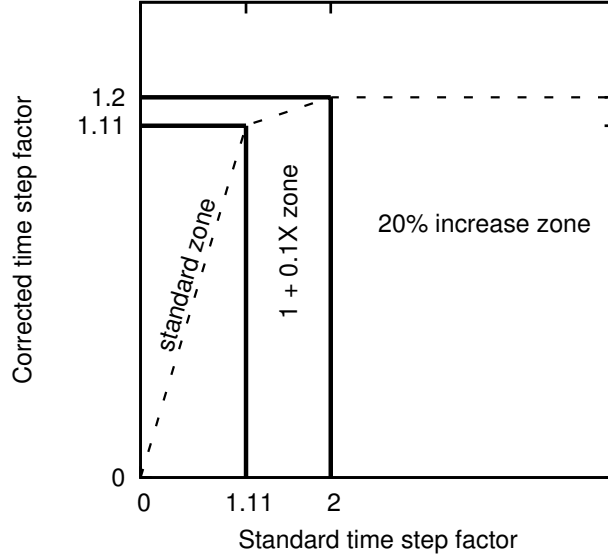
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Figure 1. Time step evolution law.

$$C = \max_{f} \left[\frac{\Delta t}{\phi V_{i|f}} \left(\frac{\lambda_n}{\lambda_t \lambda_w} |q_{w,f}| \lambda'_w - \frac{\lambda_w}{\lambda_t \lambda_n} |q_{n,f}| \lambda'_n \right. \right.$$
$$\left. \left. + (T_f \Psi) \left(|p'_{c,f}| \right) \right) \right] \leq C_{max} \qquad f = 1, N_{faces}, \quad (17)$$

where C_{max} is a user-defined limit. $V_{i|f}$ and $p'_{c,f}$ are respectively the linear interpolated values of the neighbor cell's volume and derivative of capillary pressure with respect to wetting phase saturation S_w. The C number includes all considered phenomena (gravity and capillarity) and their spatial variations to better spot local effects that could results in instability. The time-step factor F is defined as:

$$F = \frac{C_{max}}{C}. \qquad (18)$$

It can be noted that if capillary and gravity effects are neglected, Coats (C) and Todd (T) conditions reduce to the same theoretical stability restriction. This is in agreement with other analysis [24,26]. However, in practice, the Todd's number is computed from saturation variations while Coat's number is computed from fluxes. This may results in large differences in terms of time-step computations as observed in the Buckley-Leverett experiments.

2.4. *Time-step increasing factor management*

In order to improve stability and avoid large changes, the time step is computed as

$$\Delta t_{n+1} = \min \left(\min \left(F, 1 + 0.1F \right), 1.2 \right) \Delta t_n. \qquad (19)$$

where F is the timestep factor defined for each stability number (Courant, Coats or Todd). This approach is inherited from classical OpenFOAM solvers [12] . It limits the maximal increase to 20 %, and reduces the increase between $\sim$ 11 and 20 % as shown in Figure (1). Note that this heuristic management mainly occurs at the beginning of the simulations when saturation and pressure gradients are important. During simulations, the variation of stability numbers is small between time iterations and the upper bound of 1.10 % is rarely reached.

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3. Numerical experiments

In order to highlight the differences between the above stability criteria, simulations on well-known test cases are performed. We first consider a classical Buckley-Leverett experiment (viscous and gravitational regimes), then a 1D capillary rise experiment (capillary regime) and finally the 2D heterogeneous case (considering the three flow regimes) from [22]. Without other mentions, simulations are run with $k_{r,w_{max}} = 1$ and $k_{r,n_{max}} = 1$. Saturation limits are set with $S_{w,irr} = S_{n,res} = 0.001$. Brooks and Corey's m parameter is equal to $m = 5$. To assess the efficiency of stability criteria, accumulated linear solver iterations, the computational effort required, are plotted as a function of physical time of the simulated phenomenon. Indeed, the sole time step size data cannot render if the resolution is more or less time consuming. Some criterion can return a bigger time step which leads to harder to solve system for the linear solver. That is why the accumulated sum of the linear solver iterations is chosen. It provides a better idea of whether the system is fast or long to solve whatever the size of the time step is because linear solver iterations are directly proportional to CPU-time needed to inverse the matrices.

Throughout the cases, ratio between viscous flux Φ_μ, gravitational flux $\Phi_\mathbf{g}$ and capillary flux Φ_{p_c} is given if relevant. The total flux Φ_t resulting from the pressure equation (8) is considered to be decomposed as

$$\Phi_t = \Phi_\mu + \Phi_\mathbf{g} + \Phi_{p_c}, \qquad (20)$$

highlighting the competition of the different phenomena driving the flow.

3.1. Buckley-Leverett experiments

Wetting phase is injected at $|u_{inj}| = 10^{-5}$ m.s^{-1} in the same direction as gravity acceleration with an absolute scalar permeability $K = 10^{-11}$ m^2. Gradient of capillary pressure is assumed to be null. Depending on the regime, a semi-analytical solution can be calculated to predict the velocity and shape of the saturation front. This test highlights the relative permeability contribution to instability. In the gravitational case, the gravitational flux $\Phi_\mathbf{g}$ is 20 times greater than the viscous flux Φ_μ. The gravitational effects will set the front velocity.

Figures 2(a)-2(b) show the accumulated linear solver iterations necessary to reach the final physical time and highlight that the T factor is clearly too restrictive for the Buckley-Leverett case and requires between 10 and 50 times more iterations. Equation (15) gives too restrictive time steps for 1D cases as mentioned in [7]. The criteria Co and C methods have similar time-steps (Figures 2(c)-2(d)) and involve almost the same computation time. However, we should note that contrary to the C factor, the Co method is case-dependent in the setting of its upper bound Co_{max} and therefore, several tests were necessary to get the optimized value.

3.2. Capillary-gravity equilibrium experiment

In order to test the capillary pressure contribution for the different criteria, we perform simulations on a 1D vertical domain, whose lower half is filled with water (viscosity $\mu_w = 10^{-3} Pa \cdot s$ and density $\rho_w = 1000\, kg \cdot m^{-3}$). The upper half is filled with air (viscosity $\mu_w = 1.76\, 10^{-5} Pa \cdot s$ and density $\rho_n = 1\, kg \cdot m^{-3}$). The capillary pressure parameter, $p_{c,0} = 1000\, Pa$, has been tuned to balance gravity forces for this set of parameters. At the beginning of the simulation, the flow is mainly capillary-dominated due to high saturation gradients until the equilibrium state between capillary and gravity forces is reached. In this configuration, T based method efficiency is close to the Co based method (1.2 times faster) while the C method is 60 times slower with an upper bound $C_{max} = 1$ (see Figure.(3)).
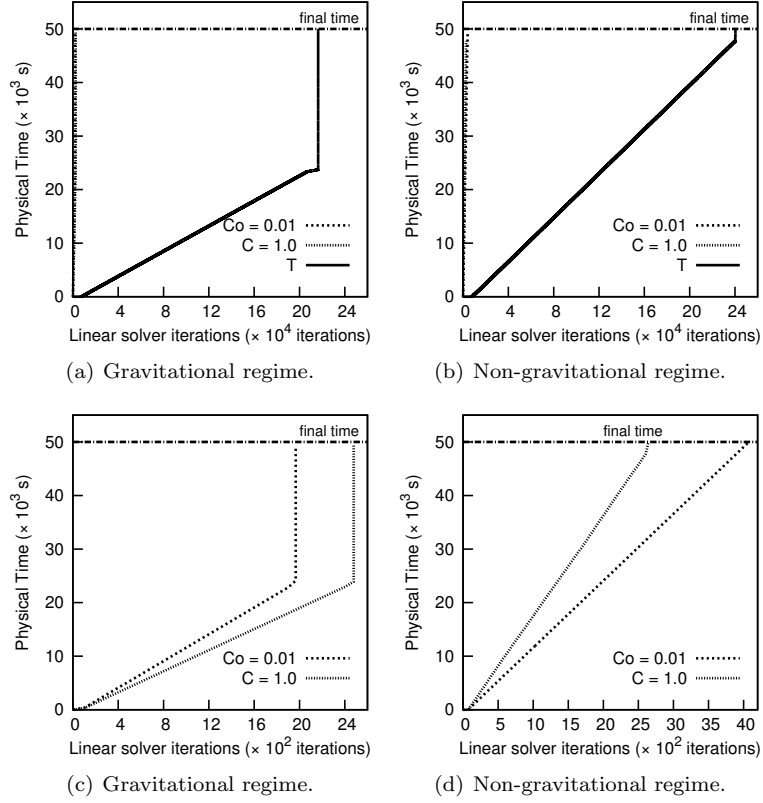
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Figure 2. Evolution of the accumulated linear solver iterations for one-dimensional Buckley-Leverett experiment using the different stability criteria T, C and Co: (a,b) all stability criteria and (c,d) focusing C and Co criteria.
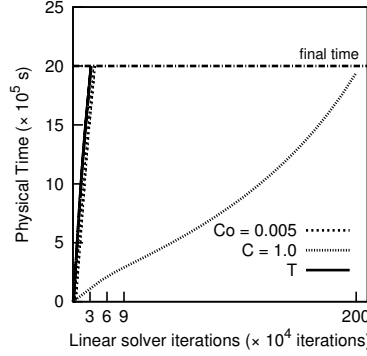

Figure 3. Evolution of the accumulated linear solver iterations for the capillary rise configuration.

The maximum stable values of parameters Co_{max} and C_{max} for the tested cases and the related maximum time-step size reached during the simulations are reported in Table 1. It can be noticed that the Coats' criteria still ensure a stable simulation with values above the standard value ($C_{max} = 1$) as observed in [8,28]. For gravitational Buckley-Leverett experiment, the saturation front is very sharp and leads to very restrictive maximum allowed time-step of similar size with Co criteria. The same remark can be derived from accumulated time steps on Figure 2(c). For capillary-gravity equilibrium simulation, even

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	Co_{max}	C_{max}	$\Delta t_{Co,max}(s)$	$\Delta t_{C,max}(s)$	$\Delta t_{T,max}(s)$
Non-gravitational Buckley-Leverett	0.15	1.19	37.49	65.25	0.18
Gravitational Buckley-Leverett	0.08	2.06	20	20.293	0.084
Capillary-gravity equilibrium	0.005	7.00	149.48	14.156	379.2

Table 1

Limits for Co_{max} and C_{max} parameters and maximum time-step allowed on Buckley-Leverett and Capillary rise cases.

though the maximal value of upper bound C_{max} is very high, the allowed time step is very restrictive. In such a configuration, Co and T criteria seem more appropriate.

3.3. *SPE 10: 2D heterogeneous case*

SPE 10th comparative solution project [22] proposed a two dimensional heterogeneous permeability field more realistic than the academic cases previously used. The different stability numbers are tested out in non-gravitational, gravitational and capillary regime. Phase densities and viscosities are given by the authors ($\rho_{gas} = 1$ and $\rho_{oil} = 700$ kg.m^{-3}, $\mu_{gas} = 10^{-5}$ and $\mu_{oil} = 10^{-3}$ Pa.s). Relative permeabilities and capillary pressure follow a Brooks and Corey law with coefficient $m = 5$. The case is an injection-production scenario: gas is injected at the left side of the domain, oil and gas are produced at the right side. Without gravity (Figure 4, top) the gas injected pushes the oil towards the production wellbore, while including gravity effects (Figure 4, middle), oil and gas segregate because of the density difference and gas overlays oil present. Capillary effects taken into account with $p_{c,0} = 0.1$ bar, tend to smooth saturation values. In order to have easily readable representation of the prescribed domain, aspect ratio 0.2:2 is adopted. The accuracy of the numerical results are ensured by considering L_2-norm error with $Co_{max} = 5 \cdot 10^{-4}$ case as reference. The configurations tested present a maximal relative error below 0.5 %. Knowing that the discretization scheme used in 2D, the T prefactor should be set to 0.25. However, gravitational case is more challenging regarding stability and T prefactor should be reduced in order to ensure stability of the simulation as mentioned in [7]. In the gravitational case, the gravitational flux $\Phi_{\mathbf{g}}$ is 100 times greater than the viscous flux Φ_μ and in capillary dominated case Φ_{p_c} is 500 times greater than Φ_μ.

The C and T methods lead to similar computation time in the non gravitational case (cf. Figure 5(a)), while Co leads to simulation 2.2 times slower.

In the gravitational case, the criterion Co leads to the fastest resolution (reported on Figure 5(b)), more than 2 times faster than C criteria. This case illustrates what has been observed before in [8,28]. C number could be either uniformly distributed throughout the domain and reaches its limit value only in one point (cf. Figure 6(a)) or has a more non uniform distribution with intermediate values and several points at the maximum value (cf. Figure 6(b)). In the first case, the stability is critical and C provides a good approximation of a suitable time step to keep the simulation stable. In the second configuration, it is remarked that C is too restrictive and stability is still ensured for values beyond $C = 1$. Due to this change in repartition, C requires almost 8 more linear solver iterations to solve the problem. The T method behaves slightly better with a 1.75 times faster simulation.

Similarly to the one-dimensional test cases, the capillary-driven case using C number is the most complex in term of stability and efficiency because it leads to unnecessary small time-steps. This is also the case for Co driven simulations because it requires to impose very low maximum limit (here 10^{-2}). Regarding efficiency, T is the best criterion as previously observed (Figure 5(c)). Comparing the efficiency for this configuration, T is 4.25 times faster than Co and 6 times faster than C.

In the case where both capillary and gravitational effects are present, the T and C criteria lead to similar performance while the Co produces a twice slower simulation (see Figure 7).
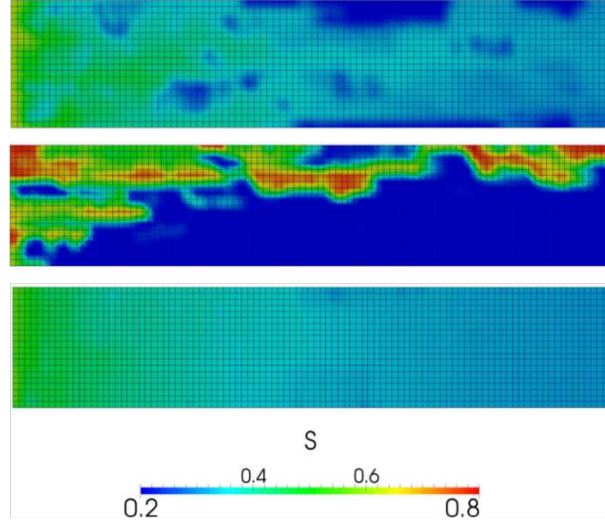
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Figure 4. Gas saturation field 2D SPE 10 case for (top) non-gravitational, (middle) gravitational and (bottom) capillary regime.
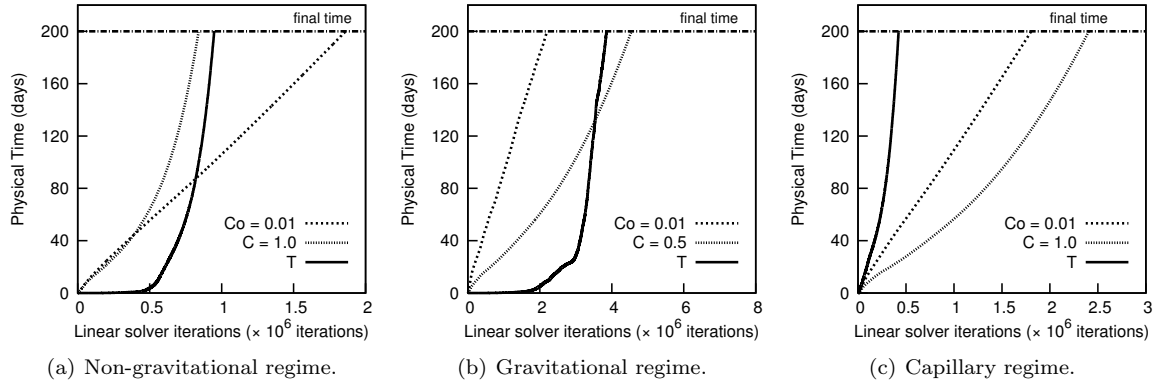

(a) Non-gravitational regime.

(b) Gravitational regime.

(c) Capillary regime.

Figure 5. Evolution of the accumulated linear solver iterations for the SPE 10 2D.
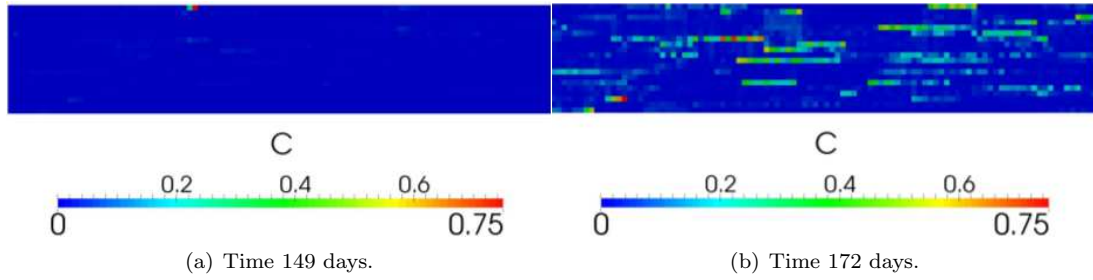

(a) Time 149 days.

(b) Time 172 days.

Figure 6. Uniform and non uniform C number distribution in gravitational regime.
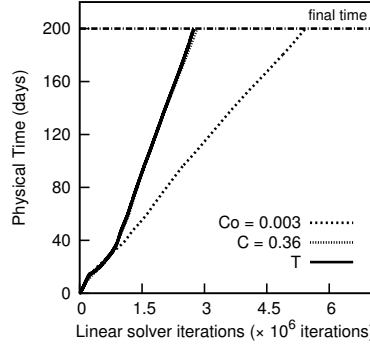
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Figure 7. Evolution of the accumulated linear solver iterations for the SPE 10 2D in capillary-gravity regime.

	Co_{max}	C_{max}	$\Delta t_{Co,max}(s)$	$\Delta t_{C,max}(s)$	$\Delta t_{T,max}(s)$
Non-gravitational	0.01	2.11	2445.3	27736	53290
Gravitational	0.01	0.56	2197	4925	7618
Capillary	0.01	3.63	2316	8583	49777
Capillary-gravity	0.003	0.36	667	1244	2512

Table 2
Limits for Co_{max} and C_{max} parameters and maximum time-step allowed on Buckley-Leverett and Capillary rise cases.

As for homogeneous porous medium, the maximum stable values of criteria C and Co are gathered in Table 2. These results hightlight that Coats number C allows the use of time-step from 2 to 10 times larger than those obtained with Co driven simulations. Non-gravitational cross-flow exhibits C_{max} larger than the similar Buckley-Leverett experiment (2.11 instead of 1.19) which can be explained because of the non-uniform distribution of maximum values of the criterion as shown in Figure 6. This configuration also occurs in capillary dominated cases. When gravitational effects are included, with or without capillary effects, $C_{max} = 1$ is no longer stable and should be reduced . Nonetheless, this latter criterion remains more efficient than Co driven simulation. The non-gravitational configuration is reported to be stable with value of $C_{max} = 2.0$ in [8].

For the challenging 2D cases that include gravity forces, Todd's criteria T has to be tuned to ensure stability. A prefactor δ is introduced as suggested in [7]. The values of this factor are respectively $\delta = 1/16$ for the gravity driven case and $\delta = 1/32$ for the capillary-gravity case.

4. Conclusion

IMPES algorithm and its sequential structure still represent an interesting alternative to coupled approaches when treating problems with a challenging number of grid cells as required by highly detailed models. However, due to the specific form of conservative equations, the derivation of stability criterion more adapted than classic CFL condition is needed. Several contributions have tried to define more adapted saturation and pressure variation criterion [7] and still studying improvement on their formulation [8].

In this study, these various criteria have been compared with the classical Courant number in various conditions leading to the following observations. For homogeneous cases C method with limit set to $C_{max} = 1$ will ensure stability in every configuration. For 2D heterogeneous cases, the C method faces

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two configuration:

— When saturation front is diffuse, which is the case for capillary or viscous dominated flows, it is safe to use limit as $C_{max} = 1$. However, it can be noted that the T method offers an interesting alternative, leading to stable and fastest simulation. An alternative approach in the capillary dominated cases is to switch to an implicit formulation, which suffer less from the loss of efficiency.

— When saturation front is sharp, which is the case for gravity and capillary-gravity cases, phases are segregated and users have to limit criterion to $C_{max} = 0.25$, in order to keep the simulations table. In the capillary-gravity configuration, the T- method leads to similar performance as the C-method.

Even if Co driven simulations can give better results in some cases, it remains a condition too dependent on the considered case. Co_{max} limit to be imposed for ensuring stability can differ by a factor of 1000 and makes this criterion unreliable. Following this work, it would be relevant to perform such a benchmark on the compressible formulation of IMPES or three-phase flow to confirm or invalidate the observations of this study.

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